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MIRROR RECORDING SYSTEM WITH A LARGE FIELD OF VIEW

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A light-gathering system consisting of four mirrors with a large field of view has been developed. Two mirrors can be combined into one unit (double mirror). The design parameters of the system have been calculated and the problem on protection of the image plane from the foreign light has been considered. The objective proposed can be used in infrared imaging as well as in photometric and optical devices operating in the infrared region of the spectrum.

Introduction. In designing infrared-imaging devices, preference is given to lenses, with which a high-quality image can be obtained by different methods in small-size systems. However, the validity of this tendency is revised every time a new optical system for the indicated devices is designed because reflective optical instruments are less expensive [1]. Mirror objectives designed for recording a weak infrared radiation of lengthy sources should have a high light-gathering power and a large angular field of view. However, in order that scanning elements could be used with such an objective, its design should be more complicated.

In [2], mirror anastigmats, described earlier in [3], are classified. These anastigmats have a compact design due to a double mirror forming their part. Analysis of the indicated systems has shown that systems of type 43-2B-II [2] can have high optical characteristics in the case where the screening, vignetting, and protecting of the image plane from foreign light are provided. All calculated variants of these systems, which can be used in practice, have a high light-gathering power, and their modifications with a parallel ray path between the second and third mirrors have a large angular field of view.

Design of an Objective. The basic diagram of a mirror recording system proposed is shown in Fig. 1. A characteristic feature of the objective is the absence of real intermediate images in the ray path behind the second mirror, i.e., $h_3 > 0$ and $h_4 > 0$. The system consists of four mirrors, two of which can be united. The first convex mirror and the second concave mirror form an afocal system, i.e., $\alpha_3 = 0$ and $h_2 = h_3$; in this case, the form of the third and fourth reflecting surfaces can be arbitrary. The second and third mirrors have central holes, through which a light beam passes. This design makes it possible to increase the field of view to $2\omega = 15^{\circ}$ as compared to analogous mirror systems and obtain an image free of spherical aberration, coma, and astigmatism in a plane field.

Calculation of the Overall Dimensions. At the first stage of calculations, we introduce the normalization condition $\alpha_1 = 0$, $h_1 = 1.0$, $f' = 1.0$, and $\alpha_5 = 1.0$ and calculate the parameters of a zero ray, determining the main design characteristics of the system considered. We have

$$\alpha_{s+1} = \frac{2h_s}{r_s} - \alpha_s , \tag{1}$$

where $h_s = h_{s+1} + \alpha_{s+1} d_s$. Let us determine the coefficient of image curvature of the third order that, when the astigmatism is corrected, meets, according to [4], the Petzval condition

$$D_0 = \frac{1}{2} \sum_{s=1}^{4} \mu_s \frac{\alpha_{s+1} + \alpha_s}{h_s} . \tag{2}$$

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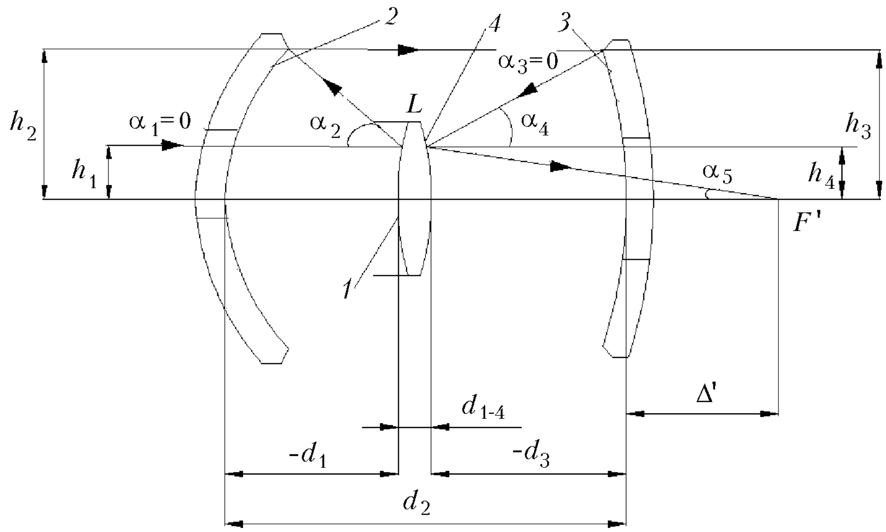

Fig. 1. Optical scheme of a mirror anastigmat [1–4) numbers of mirrors].

TABLE 1. Formulas for Reduced Values of the Design Parameters of the System

Number of surface	r_s	d_s
1	$\dfrac{2}{\alpha_2}$	$\dfrac{1-h_2}{\alpha_2}$
2	$\dfrac{2(1-\alpha_2 d_1)}{\alpha_2+\alpha_3}$	$\dfrac{h_2-h_3}{\alpha_3}$
3	$\dfrac{2(1-\alpha_2 d_1-\alpha_3 d_2)}{\alpha_3+\alpha_4}$	$\dfrac{h_3-h_4}{\alpha_4}$
4	$\dfrac{2(1-\alpha_2 d_1-\alpha_3 d_2-\alpha_4 d_3)}{\alpha_4+\alpha_5}$	—

For mirror systems in which $\mu_s = (-1)^{s+1}$ and $\mu_{s+1} = -\mu_s$, condition (2) requires that the sum of the curvatures of the convex mirrors be equal to the sum of the curvatures of the concave mirrors. The computational formulas obtained are presented in Table 1. At the second stage, we use additional relations for the axial distances:

$$d_2 = -(d_1 + d_3) + d_{1-4}\,, \quad d_3 = -(1 + \Delta') \tag{3}$$

and relations for the coefficient of central screening determined by the ray that passes near the edge of the double mirror (monounit) at a height h_1 and is incident on the third mirror (see Fig. 1):

$$\eta = \frac{h_1}{h_3 - \alpha_3 d_3}\,. \tag{4}$$

The thickness of the monounit is selected starting from the design considerations depending on the relative aperture D/f' of the objective $d_{1-4} = \left(\frac{1}{8} - \frac{1}{10}\right)\frac{h_1}{D} f'$.

It is interesting to obtain a solution with a definite coefficient of central screening in systems with a plane field. Using (2), we obtain

$$2D_0 = A_1 - (\alpha_4 + 1) = 0\,, \tag{5}$$

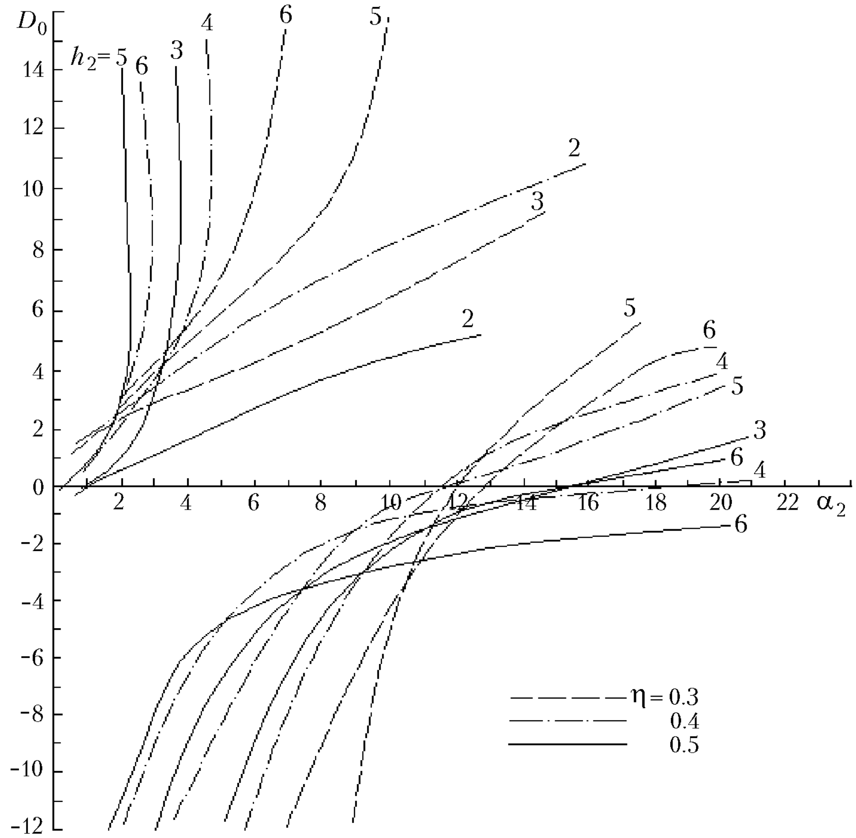

Fig. 2. Dependence of the coefficient D_0 on the parameters α_2 and h_2.

where

$$A_1 = \alpha_2 - \frac{\alpha_2 + \alpha_3}{h_2} + \frac{\alpha_4 + \alpha_3}{h_3}; \quad \alpha_3 = \frac{\eta h_2 - 1}{(d_2 - 1)\,\eta}; \quad h_3 = -\frac{\eta h_2 - d_2}{(d_2 - 1)\,\eta}.$$

The results of calculation of the coefficient D_0 at different values of the design parameters α_2 and h_2 and the coefficient of central screening η are presented in Fig. 2. Optical schemes cannot be realized in practice if the corresponding graphic dependences are discontinuous. In this case, $h_3 = 0$ and $\alpha_2 = (h_2 - 1)/[\eta h_2 - (1 + d_{1-4})]$.

To the mirror system considered, we may apply computational variants with a plane field, where $0 \le \alpha_2 \le 1$ and $\eta \ge 0.5$. In this case, the expressions for the angles α_2 and α_4 are determined from formulas (1), (3), and (5) with the use of data from Table 1:

$$\alpha_2 = -\frac{h_2^2 - (3 + \Delta')\,h_2 + 1}{(h_2 - 1)\,(1 + \Delta')}, \tag{6}$$

$$\alpha_4 = -\frac{h_2 - 1}{1 + \Delta'}. \tag{7}$$

The design parameter h_2 is determined from the inequality

$$h_2^2 - (3 + \Delta')\,h_2 + 1 < 0\,.$$

On the assumption that $Q = 3 + \Delta'$,

$$0.5\,(Q - \sqrt{Q^2 - 4}) < h_2 < 0.5\,(\sqrt{Q^2 - 4} + Q)\,. \tag{8}$$

Inequality (8) has a solution when $\Delta' > -1$ and $\Delta' < -5$ (a variant that is practically not realized). Modifications with $h_2 = h_3$ can be realized in the following regions:

$$\begin{gathered} 0.2657 \le h_2 \le 2.734 \quad (\Delta' = 0.1)\,, \quad 0.38 \le h_2 \le 2.618 \quad (\Delta' = 0)\,, \\ 0.4 \le h_2 \le 2.504 \quad (\Delta' = -\,0.1)\,, \quad 1.0 \le h_2 \le 2.25 \quad (\Delta' = -\,0.3)\,. \end{gathered} \tag{9}$$

Correction of Aberrations. In the general case, aberrations of the third order (spherical aberrations, comas, and astigmatism) in a four-mirror optical system are corrected using three surface-deformation parameters. The use of other parameters can lead to an unrealized system, a system giving a virtual image, or a system that does not transmit light to the image plane.

The equations relating the aberration coefficients of the third order to the design parameters are determined from the formulas

$$B_0 = \frac{1}{2}\sum_{s=1}^{s=4} h_s Q_s\,, \quad K_0 = \frac{1}{2}\sum_{s=1}^{s=4} h_s Q_s S_s - \frac{1}{2}\sum_{s=1}^{s=4} W_s\,, \quad C_0 = \frac{1}{2}\sum_{s=1}^{s=4} h_s Q_s S_s^2 - \frac{1}{2}\sum_{s=1}^{s=4} \mu_s \frac{\alpha_{s+1} + \alpha_s}{h_s} - \frac{1}{2}\sum_{s=1}^{s=4} W_s S_s\,, \tag{10}$$

where

$$S_s = \sum_{k=1}^{k=s-1} \frac{\mu_{k+1} d_k}{h_k h_{k+1}}\,; \quad T_s = \frac{(\alpha_{s+1} + \alpha_s)^3}{4\mu_{s+1}}\,; \quad P_s = \frac{(\alpha_{s+1} - \alpha_s)^2}{4\mu_{s+1}}(\alpha_{s+1} + \alpha_s)\,; \quad W_s = \frac{\alpha_{s+1}^2 - \alpha_s^2}{2}\,.$$

Solution of the system of equations (10) gives expressions for the quantities Q_s that determine the deformations of the mirror surfaces $\sigma_s = (Q_s - P_s)/T_s$.

From the technological considerations, it makes sense to use a double mirror with a spherical surface. On condition that $\sigma_4 = 0$, we obtain, for this variant, the system of equations

$$\sum_1^3 h_s Q_s + h_4 P_4 = 0\,, \quad -\,0.5 + \sum_2^3 h_s Q_s S_s + h_4 S_4 P_4 = 0\,, \quad A_2 - \sum_2^3 h_s S_s^2 Q_s - h_4 S_4^2 P_4 = 0\,, \tag{11}$$

where $A_2 = \sum_1^4 S_s(\alpha_{s+1}^2 - \alpha_s^2) + 2D_0$;

$$\begin{gathered} Q_1 = \frac{1}{S_3 S_2}\Big[A_2 - 0.5\,(S_2 + S_3) - h_4 P_4\,(S_4 - S_3)\,(S_4 - S_2)\Big]\,; \\ Q_2 = \frac{A_2 - 0.5 S_3 - h_4 S_4\,(S_4 - S_3)\,P_4}{h_2 S_2\,(S_2 - S_3)}\,; \quad Q_3 = \frac{A_2 - 0.5 S_2 - h_4 S_4\,(S_4 - S_2)\,P_4}{h_3 S_3\,(S_3 - S_2)}\,. \end{gathered} \tag{12}$$

First Variant. We now consider one of the possible variants of an objective with a fourth plane mirror, where $\alpha_4 = -\alpha_5 = 1.0$, $h_2 = 2.0$, and $\Delta' = 0$, which corresponds to (9). In this variant, the first three mirrors are deformed and formulas (12) are simplified:

$$Q_1 = -\,h_2\,(Q_2 + Q_3)\,, \quad Q_2 = \frac{0.5 S_3 + A_2}{h_2 S_2\,(S_3 - S_2)}\,, \quad Q_3 = -\frac{0.5 S_2 + A_2}{h_2 S_3\,(S_3 - S_2)}\,. \tag{13}$$

TABLE 2. Design Parameters of Two Variants of Objectives (f = 200 mm)

Number of surface	r, mm	d, mm	σ	Shape of mirror surface
1	400.0	–200	0.925928	Flattened
	400.0	–160	2.320618	Ellipsoid
2	800.0	420	–1.04878	Hyperboloid
	666.7	300	–0.231182	Ellipsoid
3	–400.0	–200	0.011744	Flattened sphere
	–550.8	–120	0.090302	With a small deformation
4	∞	—	0	Plane
	–1200.0	—	0	Sphere

Here, the coefficient of central screening η = 0.5. The linear value of the field of view is determined by the height of the ray screened by the third mirror

$$y' = \frac{\eta h_3}{2D} f' . \tag{14}$$

The calculated angular field of the system, according to formula (14), can be large.

The objective was calculated for the focal distance f' = 200 mm, the relative aperture D/f' = 1:1, and the angular field of view $2\omega = 15^{\circ}$.

Second Variant. For an objective with an image plane positioned outside the system, the range of possible values of h_2 is narrower ($h_2 > 1$). The calculations were performed for $\Delta' = -0.3$. The range of solution of this variant of the system is obtained from condition (9) at $\Delta' = -0.3$. The transverse and longitudinal dimensions of the objective, which are smaller than those in the first variant (h_2 = 1.8, $d_2 = 1.5f'$), provide, for the same values of the focal distance and the relative aperture, a field of view of the mirror system $2\omega = 12^{\circ}$.

The design parameters of the two objectives, calculated in the region of aberrations of the third order, are presented in Table 2. As a result of the further optimization of the variant ($\Delta' = -0.3$), we obtained a recording mirror system with a third and fourth spherical mirror, which is of practical interest. The shapes of the nonspherical surfaces of the first and second mirrors of this system are determined from the equations

$$y^2 + z^2 = 800x - 2.96061x^2 - 0.0148328x^3 , \quad y^2 + z^2 = 1333.34x - 0.484758x^2 .$$

Calculation of the Lens Screen and the Vignetting. In the recording mirror system considered, provision is made for the protection of the image plane from both direct and secondary light foreign at a minimum vignetting. Since the image plane is positioned practically at infinity when the mirror barrels are projected into the space of objects positioned in the path of the parasite ray reflected only from the third and fourth mirrors, the possible foreign rays coming from the system of the first two mirrors at the angle of the field of view are considered. In this case, the most efficient method of protection from the foreign rays is increasing the diameter of the first mirror, which makes it possible to introduce a lens screen L (see Fig. 1); however, this somewhat increases the coefficient calculated by formula (4). The vignetting is equal to 30° for the angular field of view $2\omega = 12^{\circ}$ at η = 0.57, which is allowable for light-gathering systems.

Conclusions. The system proposed provides a stable correction of aberrations in the field of view: in the setup plane, the circle of confusion is equal to 0.1 mm at the center and does not exceed 0.05 mm, with a small background exceeding this limit, at the edge. The image curvature is corrected and the astigmatism does not exceed 0.05 mm. At the edge of the field of view, the frequency 60 mm^{-1} is reproduced with a contrast of no less than 0.5. Mirrors for the objective proposed should be fabricated and aligned with the following technological tolerances: minimum turn of mirrors, from $\pm 1'$ to $\pm 2.5'$; decentering, from ±0.1 to ±0.5 mm. The absence of chromatic aberrations, the high resolving power, and the acceptable conditions for disposition of receiving apparatus in the recording mirror system proposed allow it to be widely used. The results of our calculations and the formulas obtained can be used for designing new mirror systems for heat scanning as well as radiometric and direction-finding devices.

NOTATION

A_1, A_2, auxiliary variables in formulas (5), (11)–(13); B_0, C_0, D_0, coefficients of monochromatic aberrations of the third order (spherical aberration, astigmatism, image curvature); D, diameter of the entrance pupil, mm; D/f', relative aperture; d_s, axial distances from the top of the previous mirror surface to the top of the next mirror surface; F', back focus of the objective; f', back focal distance of the objective, mm; d_{1-4}, thickness of the double mirror; h_s, height of the zero objective ray on the main planes of the mirror surfaces; K_0, coefficient of monochromatic aberration of the third order (coma); n_s, refractive index of the optical medium before the mirror surface; r_s, radius of the mirror surface; S_s, T_s, P_s, W_s, Q_s, auxiliary quantities of aberration calculation of the third order; $Q_4 = P_4$ ($\sigma_4 = 0$); Q, auxiliary variables in formula (8); x, y, z, surface coordinates in the Cartesian system; y', linear dimension of an image; α_s, tangent of the angle formed by the zero objective ray and the optical axis before the mirror surface; α_{s+1}, tangent of the angle formed by the zero objective ray and the optical axis behind of the last mirror surface of the system in the image space; Δ', value of the displacement of the radiation-detector plane relative to the top of the third mirror; η, linear coefficient of central screening; $\mu_s = 1/n_s$, reciprocal of the refractive index n_s; μ_{s+1}, reciprocal of the refractive index n_{s+1} determining the optical medium of the image space; σ_s, deformation of the mirror surface; ω, angle of the field of view. Subscripts: s, number of mirror surface (s = 1, 2, 3, 4); 0, zero index of the monochromatic-aberration coefficient corresponding to the entrance-pupil position coincident with the first surface.

REFERENCES

1. J. M. Lloyd, *Thermal Imaging Systems*, Plenum, New York (1975).
2. N. K. Artyukhina and A. P. Shkadarevich, Classification of four-mirror anastigmats with a double mirror, *Dokl. Nats. Akad. Nauk Belarusi*, **49**, No. 1, 41–45 (2005).
3. V. N. Churilovskii and N. K. Kudrina, Some types of four-mirror objectives, *Izv. Vyssh. Uchebn. Zaved.*, *Priborostroenie*, **20**, No. 7, 93–97 (1977).
4. V. N. Churilovskii, *The Theory of Chromatism and Aberration of Third Order* [in Russian], Mashinostroenie, Moscow (1968).